Physics-Informed Neural Networks with Application in Computational Structural Mechanics

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Abstract

Structural mechanics is ubiquitous in the mechanical, aerospace, civil, and biological sciences. With the advent of machine intelligence technology, it is imperative to study computational structural mechanics from a machine intelligence standpoint. Helicopter rotor blades are simulated as cantilever beams, and their complete mechanical behaviour understanding, such as deflections under various loading conditions, is critical for rotorcraft mechanics. This investigation studies the deflection of beams using state-of-the-art physics-informed neural networks (PINNs). An Euler-Bernoulli beam theory-based beam deflections analytical solution is compared with the PINNs solution. A good convergence is observed between PINNs and closed-form results. We would like to extend this work to Timoshenko beam theory and other dynamical systems in computational structural dynamics (CSD) to develop a digital twin for aerospace and mechanical sciences systems.

Keywords Physics Informed Neural Networks · Computational Structural Mechanics · Neural Networks · Solid Mechanics · DeepXDE

1 Introduction

Physics-informed neural networks (PINNs) have emerged as a groundbreaking tool blending deep learning and physical modeling in the ever-evolving engineering and computational science landscape. PINNs represent a novel paradigm in which neural networks are trained to solve supervised learning tasks while respecting the underlying physical laws described by general nonlinear partial differential equations[1, 2]. This fusion of physics-based constraints with the versatility of neural networks opens new avenues for solving complex engineering problems, particularly in aerospace engineering.

This paper primarily focuses on applying PINNs in beam mechanics, a fundamental area in structural engineering with profound implications in aerospace design and analysis[1-4]. Traditionally approached through analytical and numerical methods, beam mechanics stands to gain significantly from integrating PINNs, offering enhanced accuracy and efficiency in simulations[5-6]. The versatility of PINNs, particularly in embedding physical laws into the learning algorithm, ensures that the solutions are not just data-driven but also explainable. This aspect is crucial in aerospace engineering, where adherence to physical realities is non-negotiable.

Rotorcraft researchers are continuously exploring novel and computationally efficient algorithms to solve critical aeromechanics problems, such as helicopter vibration reduction using smart trailing edge flaps at the rotor blades[7]. Rotor blades can be simulated as cantilever beams for modeling and simulations. There is a constant quest worldwide to develop and integrate novel computational methods to advance rotor mechanics further and computational structural mechanics.

The primary objective of this investigation is to explore the potential of PINNs in various aspects of aerospace engineering. From structural analysis and aeroelasticity to thermal management and propulsion system optimization, PINNs offer a versatile tool for tackling complex problems that are otherwise challenging to solve with traditional methods. In aerospace engineering, where the systems are often subjected to extreme conditions and complex interactions, PINNs can provide more accurate, efficient, and robust solutions.

2 Introduction to the Idea of Physics Informed Neural Networks (PINNs)

From a mathematical perspective, PINNs[2] can be comprehensively appreciated by recognizing that the neural network's output is systematically amalgamated with the residual of the system's differential equation and corresponding boundary conditions, resulting in a precise solution. This formulation extends the foundational principle of neural networks, wherein the input features are linearly transformed through weights and biases, thereby integrating the physical laws governing the system into the learning architecture.[1]. It can be thought of two parameters, where the weight is simply the variable parameter, which is what is associated with the input, while the biases are constant values which is what the network adds to the Input before passing it onto the Activation Function.

$$\sum Weight * Inputs + Bias$$

The key differentiation between the PINNs methodology and conventional neural network paradigms principally manifests within the framework of the loss function. In PINNs, the composition of the loss function is intricately expanded to include not only the Mean Squared Error (MSE) loss, typical of standard neural networks but also incorporates the physics loss, i.e. boundary conditions loss and the derivative loss. This enriched loss function undergoes an iterative optimization, a process elegantly illustrated in Figure 1 below.

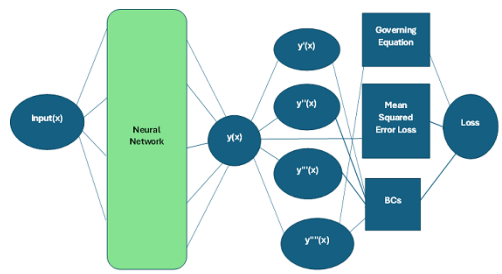

Figure 1: The structure of a Physics Informed Neural Network

The network ingests spatial inputs x, applies a feedforward neural network (FNN) with three hidden layers and 20 neurons each, using the hyperbolic tangent activation function and Glorot normal initialization. Along with the mean squared error loss, the PINNs Network also takes into account the Boundary Conditions, the Governing Equation, for which the Network uses the 1st, 2nd, and 3rd derivates as well.

3 Physics Explanation

In this study, a cantilever beam, subjected to a uniform distributed load (an analogous special load case of a fixed wing or rotary wing), illustrated in Figure 2, was considered for the PINNs case-study [8]. Over here, we take L as the Length of the Beam. To get the governing equation,

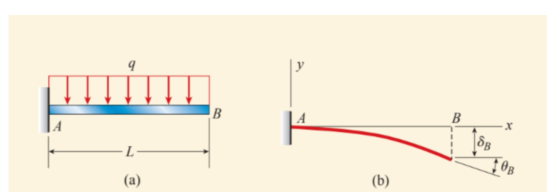

Figure 2: Cantilever Beam with Uniformly Distributed Load [8]

the bending moment in the beam at distance x (the point's location along the beam) from the fixed support can be calculated as,

$$M = \frac{-qL^2}{2} + qLx - \frac{qx^2}{2} \tag{1}$$

Where q is the uniform load intensity. The differential equation v" for beam deflection according to the Euler-Bernoulli theory is :

$$v'' = -\frac{M(x)}{EI} \tag{2}$$

M(x) is the bending moment along the beam, E is our Elastic Modulus, which indicates how stiff the beam is, and I is the 2nd Moment of Inertia.The first equation can be plugged into the beam deflection curve differential equation[8], and the combined equation is:

$$EIv'' = \frac{-qL^2}{2} + qLx - \frac{qx^2}{2} \tag{3}$$

Integrating the above equation and using the boundary conditions, the angle of rotation v' is calculated as follows:

$$v' = \frac{-qx}{6EI}(3L^2 - 3Lx + x^2) \tag{4}$$

The following boundary conditions are considered:

$$v(0) = 0, \quad v'(0) = 0 \tag{5}$$

$$v'(L) = \frac{-qL^3}{6EI} \tag{6}$$

$$v(L) = \frac{-qL^4}{8EI} \tag{7}$$

Upon integration for the deflection equation with the boundary conditions, which is the equation compared to the real solution, the equation for the deflection of this beam is:

$$v = \frac{-qx^2}{24EI}(6L^2 - 4Lx + x^2) \tag{8}$$

4 Results and Discussion

In this study, a beam measuring 2.7 meters long was considered and is subjected to a uniform load of 60 kilo-Newtons, Young's Modulus is equal to:

$$200 \times 10^9 \text{Pa} \tag{9}$$

and Moment of Inertia is equal to:

$$0.000038929334 \quad \text{kg} \cdot \text{m}^2 \tag{10}$$

With the geometric parameters and PINNs formulation in place, the exact solution and PINNs solution is illustrated in Figure 3, where it can be observed that the PINNs network reproduces exactly the same solution as the exact solution of the Equation.

Before reaching the final result, a comprehensive parametric study is conducted to identify the best number of epochs for convergence and different activation functions are also explored, such as ReLU, tanh, ELU, Sigmoid, and Softmax. Various optimizers are also explored, such as Adam, RMSProp, and Stochastic Gradient Descent, and different learning rates are also investigated as part of the parametric study. The result finally matched the exact Solution of the Cantilever beam equation at 2000 epochs, using the Adam optimizer and the tanh activation function with a learning rate of 0.01 with three hidden layers in the Code.

It can be concluded that PINNs are capable of fast and accurate computations for computational mechanics problems and would be relevant to aerospace structures, other mechanical sciences geometric primitives, and industrial systems. In the future, these systems are capable of being at the forefront of the industry, and we can prove through this research that any PINNs network will reproduce an accurate solution for any system if calibrated properly.

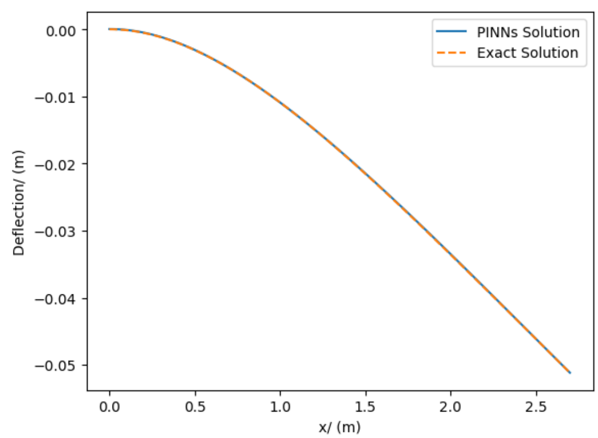

Figure 3: The Graph of the PINNs Solution vs the Exact Solution

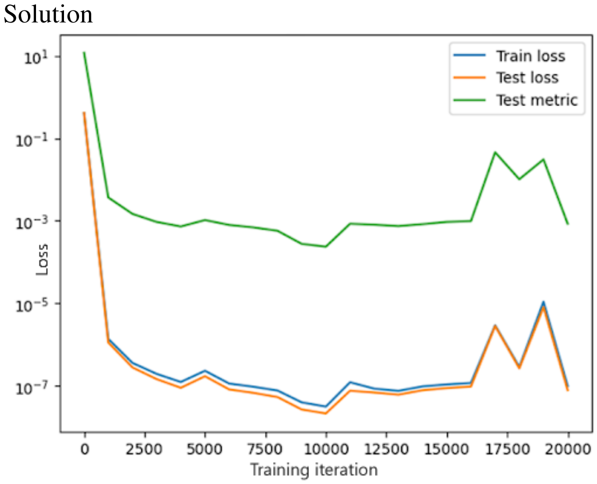

Figure 4: Convergence of the PINN model over 20,000 training iterations, showing the decrease in training and test losses with the L2 relative error as a test metric.

In Figure 4, we present the convergence trends of the Physics-Informed Neural Network (PINN) over 20,000 training steps (epochs). The blue line represents the training loss, and the orange line represents the test loss. Both losses demonstrate a steep decline in the initial training phase, followed by a plateau, indicating the model's learning and subsequent stabilization. The green line represents the test metric, which predicts how different the real values are from the predicted values of the network, specifically the L2 relative error, which is a normalized measure of the model's predictive accuracy across the test dataset[6].

The L2 relative error is mathematically defined as:

$$L2 \, \text{relative error} = \frac{\sqrt{\sum_{i=1}^{N} (y_{\text{pred},i} - y_{\text{true},i})^2}}{\sqrt{\sum_{i=1}^{N} y_{\text{true},i}^2}} \qquad (11)$$

where N is the number of samples in the test dataset, $y_{\text{pred},i}$ is the model's prediction for the i-th sample, and $y_{\text{true},i}$ is the corresponding true value. The L2 relative error quantifies the overall deviation of the model's predictions from the true values, normalized by the magnitude of the true values to facilitate comparison across different scales.

In the context of Figure 4, the declining trend of the L2 relative error alongside the losses reflects the model's increasing accuracy. Notably, the green line exhibits fluctuations, particularly in the latter half of the training process. These fluctuations in the L2 relative error could be indicative of variations in the model's performance on the test dataset at different iterations and warrant further investigation to ensure the robustness and reliability of the predictions.

5 Conclusion

In this investigation, the state-of-the-art physics-informed neural networks (PINNs) technique is studied for applications in computational mechanics, in particular beam mechanics. A good convergence is observed between PINNs, and closed-form results of the Euler-Bernoulli theory-based beam deflection analytical solution. PINNs solutions are also computationally efficient; the results took an average of 19.5 seconds to come up with a solution to the equation, proving that the Physics Informed Neural Networks (PINNs) give effective and reasonably fast results for mechanical and aerospace systems with optimal hyperparameters. We used 100 residual points for testing the residual and fixed 32 residual points for training, which proved effective, with 3.71e-08 being the best train loss and 2.72e-08 being the best test loss. Integration of data-driven and physics is an ideal candidate to develop a digital twin for aerospace and mechanical sciences systems. In future, this work will be extended to Timoshenko beam theory and other dynamical systems in computational structural dynamics (CSD) and also develop an optimization routine for automatic hyperparameter tuning.

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