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Advancing Structural Vibration Analysis: Implementation of PINNs for Aerospace Applications

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Abstract

With the increase in complexity, partial differential equations (PDEs) solutions have become challenging to solve numerically. Hence, new techniques, such as PINNs, are currently being used and implemented. PINNs, being neural networks trained to fit the physics of problem-solving partial differential equations and being helped by the complex architecture of neural networks, are valuable tools for solving complex problems. This paper aims to bring the usage of Physics Informed Neural Networks to light by using it to solve the 1D axial bar vibration equation. The equation has been compared and validated, with the analytical solutions generated through numerical methods and a finite element method solution. The graph shows good convergence to the results. The work shows the practical usage of the PINNs for the computational requirements of aerospace engineering simulations, which would work as a base for further development and refinement of how we could accurately find a solution to a physics problem. The additional work aims to bring biaxial bending case for the solution through PINNs, which could be implemented and compared due to the high complexity of their analytical solutions.

Keywords PINNs · Neural Networks · Physics Informed Neural Networks · Axial Vibration · Solid Mechanics · DeepXDE

1. Introduction

The emerging concept of physics-informed neural networks is a methodology that combines the deep learning regime and physical modeling together to drive further breakthroughs in engineering and computational science [1]. PINNs offer a unique way of training neural nets with supervised learning problems and are compatible with the most fundamental physical principles described by general non-linear partial differential equations. Different constraints based on Physical Laws, coupled with the transformative capability of neural nets, are used to solve complex engineering problems, especially in aerospace engineering [2], [3].

Bar vibration problems [4] have been used as a base problem in the case of the development of wings for the simulation of wings as a bar with axial loads to find the effect of axial load, which acts on the wing because of aerodynamic loads in total, which might cause fatigue in the structure, creating cracks and elongation [5]. These could also be used extensively to solve problems arising across similar structures in the aircraft's airframe. The analysis can be used as a base case for monitoring and optimizing structural health design [6].

The research findings discussed here focus mainly on the applicability of PINNs in multiple areas of aerospace engineering. In the field of aerospace, many structural applications and simulations can greatly benefit from this technology. This could be applied in a very efficient and robust manner for solving very complicated problems that are extremely challenging for traditional approaches. PINNs is a framework that is being implemented to solve various complex problems ranging from the modeling of a tumor cell [7] to the structural health monitoring of an Aircraft [8]. Being in research, its future promises great value in addition to solving the complex problems in aerospace engineering that are currently being solved with great difficulty [9]. Development of the models to solve complex problems such as Structural health monitoring is currently under research.

2. PINNs Framework

PINNs are neural networks whose architecture is systematically derived from the boundary conditions and the residual of the system boundary conditions. This gives a highly accurate solution. Inputs into the network are passed through weights and biases[1] followed by activation functions within the network while incorporating physical laws within the system. Integrations are achieved by backpropagating through gradient descent. Every neuron in the structure contains two parameters: a weight and a bias. Later, all this information gets passed through the activation functions, an optimizer is utilized, and weight optimization is done with the help of backpropagation[1].

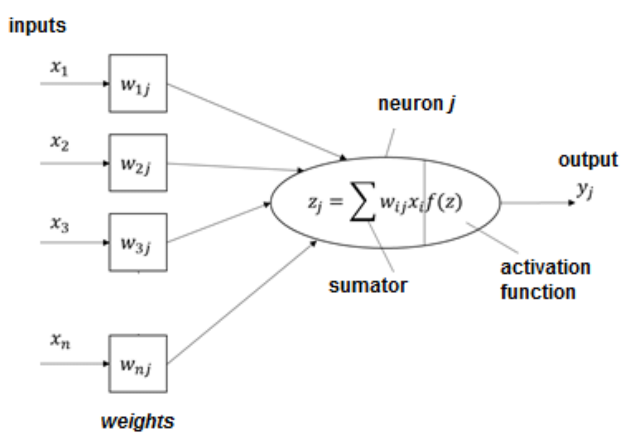

Figure 1: Architecture of PINNs

The key difference between PINN methodology and conventional neural networks is the integration of physics loss[1]. Physics loss includes the losses generated due to boundary conditions and losses generated due to the differential equation. The Physics Loss is then merged with the data loss (if available through experimental data) and iterated over the neural network. This is why PINNs do not need a lot of data to work with and find accurate solutions to problems even without experimental data from the outputs. The architecture and the backpropagation for optimization are shown in Figures 1 and 2, respectively.

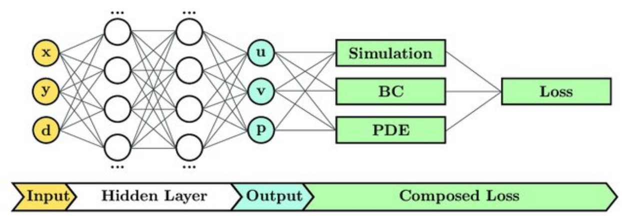

Figure 2: Architecture of PINNs

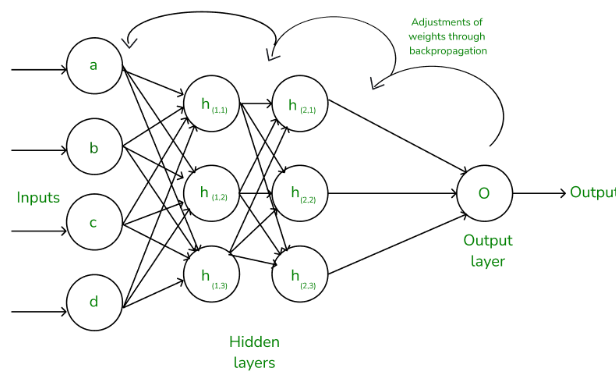

Figure 3: Updation of weights through Backpropagation

We are utlising the DeepXDE [10] library for easier and faster computation of PINNs solutions. The library has in-built functions developed and tested for faster computation. The library is still in research and constantly evolving to merge usecases from PINNs to DeepONets [10]. The model takes space-time points (x,t) and applies them via a neural network with 5 hidden layers, each with 40 neurons, with the hyperbolic tangent activation function and Glorot uniform initialization. It also contains boundary conditions and equations by reducing the mean square error loss in the PINNs model by some criterion testing of the first and second derivatives .

3. Physics behind 1D Axial Vibration

In this study, a bar is considered for the PINN case study under fixed-fixed conditions at the boundaries, subject to a uniform load that varied as cos (x) applied axially, illustrated in Figure 3. Here, we take L as the length of the bar and EI as its characteristics. To get the governing differential equation, we have

$$\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = \frac{q(x)}{\rho A} \tag{1}$$

$$q(x) = cos(x) \tag{2}$$

The following boundary conditions and initial conditions were considered for our problem:

$$u(0, t) = 0, u(L, t) = 0 \tag{3}$$

$$u(x, 0) = 0, \frac{du}{dt}(L, 0) = 0 \tag{4}$$

The solution is broken into two parts, a homogeneous solution and a particular solution to the problem. Applying the boundary and initial conditions, the final analytical solutions come out to be as:

$$u_{particular} = cos(x) + x(1 - cos(1)) - 1 \tag{5}$$

$$u_{homogeneous} = \sum (A_n cos(n\pi t) sin(n\pi x)) \tag{6}$$

where A_n is the Fourier coefficient required for expanding the series, L is the length of the bar and ρ is the density of the bar, A is the Cross-sectional Area of the bar and c is the wave speed.

To solve these equations using the PINNs, the loss function is defined as follows:

$$Loss_{equation} = \frac{\partial^2 NN}{\partial t^2} - c^2 \frac{\partial^2 NN}{\partial x^2} - \frac{q(x)}{\rho A} \tag{7}$$

$$Loss_b = (NN(0, t) - 0)^2 + (NN(L, t) - 0)^2 \tag{8}$$

$$Loss_i = (NN(x, 0) - 0)^2 + (\frac{\partial NN}{\partial t}(L, 0) - 0)^2 \tag{9}$$

where NN is the output of the network being reiterated for the loss to be used in the model and Boundary losses and Initial condition losses are given as in Equation (8) and Equation (9) respectively. The general usage of mean squared error is done if the data loss is available for the specific usecase. As in this research no data has been generated, we are using only the physics of the model and solving the problem through PINNs.

4. Results and Discussion

In this study, a bar measuring 1 m long was considered. Young's modulus and Moment of Inertia is equal to

$$\rho = 7850, A = 0.01, c = 32.44 \tag{10}$$

The load is applied axially and uniformly, varying as

$$q(x) = cos(x) \tag{11}$$

The bar is in fixed-fixed condition on both ends. With the boundary and initial conditions defined along with the general differential equation, the exact solution and PINN solution for the specific case is illustrated in Figure 6, where it can be observed that the PINN network converges similarly to the analytical solution of the equation. There are dissimilarities due to various factors and tuning. The exact plot the DeepXDE Library generates is shown in the Figure 5, which shows $u(x, t)\ vs\ x\ and\ t$, and Figure 4 shows the training and test loss for the model. This work will be extended further to solve the load variation in x and time and get better converging results.

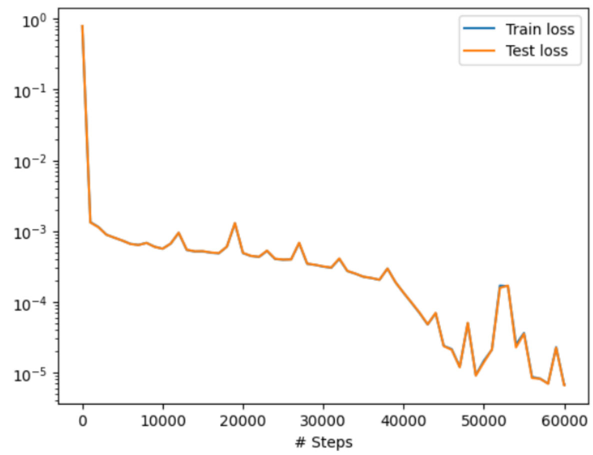

Figure 4: Training and Test Loss

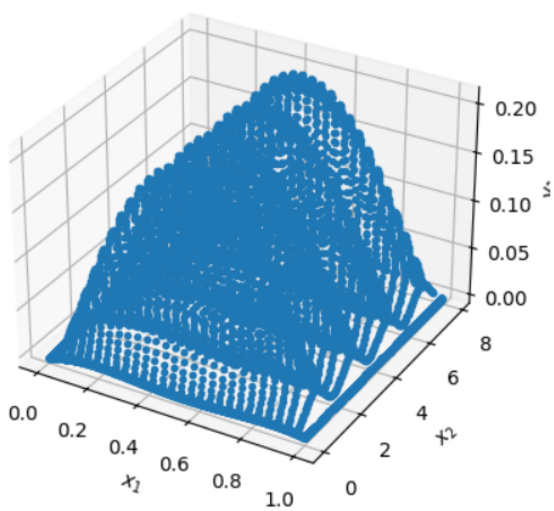

Figure 5: 3D plot for the solution of PINNs

To get the final result, a large parametric experiment was run to figure out how many epochs are needed to achieve convergence. We evaluated different activation functions (ReLU, tanh, ELU, sigmoid, and SoftMax). Multiple optimizers, including Adam, RMSProp, and Stochastic Gradient Descent, were also explored, as well as different learning rates. The results were found to be close to the exact solution. We are also trying to tune the model for very accurate convergence. We reached the final convergence with 60000 epochs, Adam optimizer, and tanh activation function of learning rate 0.01 and 5 hidden layers in the code.

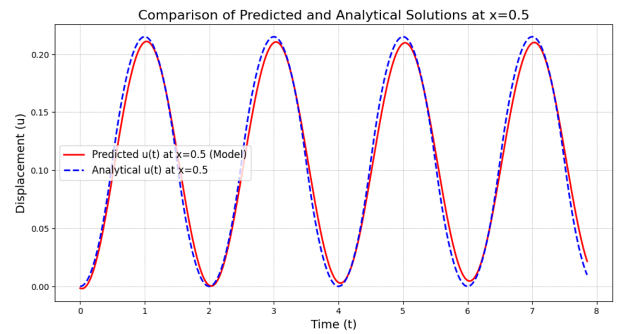

Figure 6: Comparison plots of u(t) at a fixed x

It shows that PINNs perform well for fast, reliable computations of computational mechanics problems. They also apply to aerospace architectures, other mechanical sciences, geometric antecedents, and manufacturing machinery. If calibrated, PINNs can confidently generate accurate solutions for any system and are an attractive tool for future applications in these fields. Moreover, PINNs can be used to solve time- and space-varying problems that traditional methods do not suit. This study is an important development in solving difficult equations in aerospace, leading to novel algorithms for dynamical systems and offering a clear advantage over traditional analysis methods.

5. Conclusion

In this paper, we have solved the 1D axial bar vibration problem using PINNs, which involved variance along spatial as well as time domains in the governing differential equation. We have obtained a good convergence of the solution obtained through the model and the analytical solution. This reflects the potential of PINNs in the aerospace industry. Through this research, the PINNs are not establishing the improvement as, in this case, PINNs are less efficient than the analytical solutions but could be improved and used to develop the models where the solutions are difficult to obtain. In this research, we have used

15000 boundary points and 5000 test points with a learning rate of 0.01 and 60,000 epochs. This proved to be effective for our model but could need tuning for the change of parameters or the conditions in the GDE. We have achieved a minimum loss of 1.12e-05 for this case. The integration of the Physics of the phenomena and the experimental data would help us develop an exact replica of the solutions with the required accuracy. This work will be extended to bi-axial bending and other cases of axial vibration in the future. An Optimisation routine for hyperparameter tuning is also to be developed.

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